GCE AS/A level

0981/01

MATHEMATICS M2
Mechanics 2

P.M. THURSDAY, 21 June 2012

1½ hours

0981
010001

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Take g as $9{\cdot}8\,\text{ms}^{-2}$.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A particle moves in a straight line with velocity $v\,\text{ms}^{-1}$ at time $t\,\text{s}$, where

$$v = 4\cos 2t.$$

Calculate the distance travelled by the particle between $t = 0$ and $t = \frac{\pi}{6}\,\text{s}$. [3]

2. One end of a light elastic string, of natural length $\frac{5}{3}\,\text{m}$ and modulus of elasticity 245 N, is attached to a fixed point O. The other end of the string is attached to a particle of mass 7·5 kg. The particle hangs in equilibrium vertically below O.

(a) Calculate the extension of the string. [3]

(b) Determine the elastic energy stored in the string. [2]

3. A particle moves on a horizontal plane so that at time t seconds its position vector **r** metres relative to a fixed origin O is given by

$$\mathbf{r} = (t + 2t^2)\mathbf{i} + (1{\cdot}5t^2 - 2t)\mathbf{j}.$$

(a) Determine the time when the velocity of the particle is perpendicular to the vector $(-\mathbf{i} + 2\mathbf{j})$. [5]

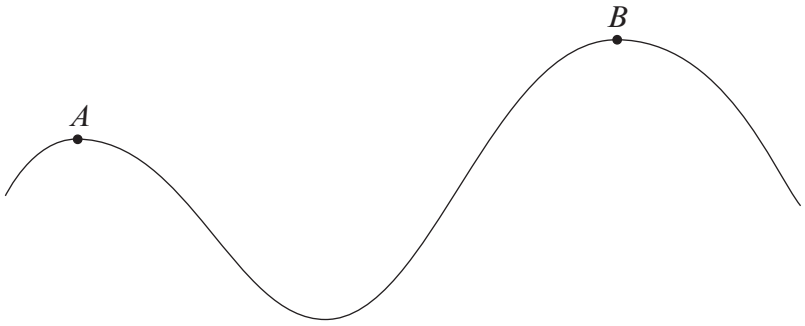
(b) Show that the acceleration of the particle is constant and find its magnitude. [3]

4. A vehicle of mass 1200 kg is moving up a slope inclined at an angle of α to the horizontal, where $\sin\alpha = 0{\cdot}1$. The resistance to motion is modelled as a constant force of magnitude 600 N.

(a) The vehicle's engine is working at the rate of 75 kW. Calculate the magnitude of the acceleration of the vehicle when its velocity is $25\,\text{ms}^{-1}$. [5]

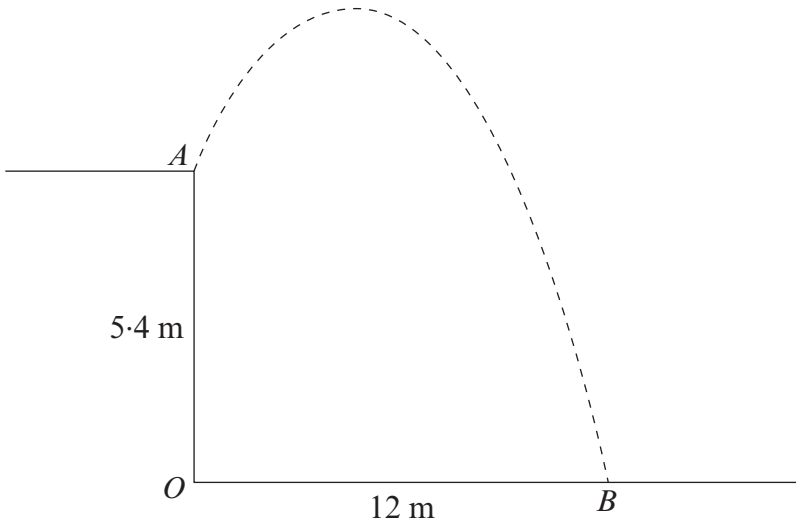
(b) When the vehicle's engine is working at the rate of 90 kW, calculate the constant speed which can be sustained by the vehicle. Give your answer correct to 3 significant figures. [4]

5. The diagram shows two points A and B on a track. A toy car of mass 0·1 kg travels on the track from A to B.



The heights of A and B above floor level are 0·5 m and 1·4 m respectively.
The length of the track between A and B is 1·2 m. The resistance to motion of the toy car is assumed to have a constant magnitude of 6 N. The toy car is given a velocity of v ms^{-1} at A and comes to rest at B. Calculate the value of v.
Give your answer correct to 3 significant figures. [7]

6. A pebble is projected from a point A which is 5·4 m vertically above a point O on horizontal ground.



The initial velocity of the pebble is V ms^{-1} at an angle α above the horizontal, where $\tan\alpha = \frac{3}{4}$.
The pebble hits the ground at the point B which is at a distance of 12 m from O.

The time of flight of the pebble is T s.

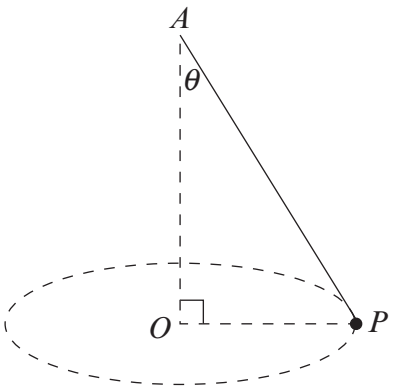
(a) Write down the horizontal component and the vertical component of the initial velocity of the pebble in terms of V. [2]

(b) Show that $VT = 15$. [2]

(c) Find the value of T and hence find the value of V. [4]

(d) Determine the speed of the pebble as it hits the ground at B. [5]

7. One end of a light inextensible string is attached to a fixed point A. The other end is attached to a particle P of mass 3 kg. The point O is vertically below A and P moves in a horizontal circle of centre O with a uniform angular speed of 2·8 radians per second. The tension in the string is 88·2 N and $O\hat{A}P$ is θ.



(a) Find the value of θ. [3]

(b) Calculate the length of the string. [5]

8. A ship S is moving in a straight line with constant velocity. At time $t = 0$, its position vector relative to a fixed origin O is $(8\mathbf{i} + 7\mathbf{j})$. At time $t = 3$, its position vector is $(14\mathbf{i} - 5\mathbf{j})$.

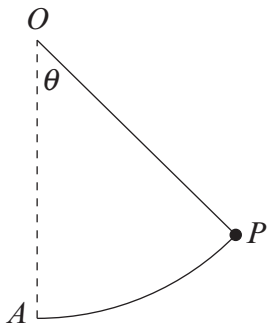
(a) Show that the velocity of S is $(2\mathbf{i} - 4\mathbf{j})$. [2]

(b) Find an expression, in terms of t, for the position vector of S at time t. [2]

At time $t = 10$, a boat B leaves O and travels with constant velocity $x\mathbf{i} + y\mathbf{j}$, intercepting S at time $t = 50$.

(c) Calculate the value of x and the value of y. [6]

9. A particle of mass 3 kg is attached to one end of a light inextensible string of length 1·2 m. The other end of the string is attached to a fixed point O. Initially, the particle hangs vertically below O at the point A. The particle is then projected horizontally with speed u ms^{-1} from A. When the particle is at the point P, the string makes an angle θ with the vertical OA as shown in the diagram.



The particle comes to instantaneous rest when $\cos\theta = \frac{2}{3}$.

(a) Calculate the value of u and find an expression for v^2 in terms of $\cos\theta$, where v is the velocity of the particle at P. [6]

(b) Find an expression, in terms of θ, for the tension in the string when the particle is at P. [4]

(c) Determine the greatest value and the least value of the tension in the string. [2]

